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Tunnel transitions, vacuum background and scattering processes in quantum electrodynamics†

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Abstract. It is shown that in external fields there may occur an excitation (deformation) of the vacuum state of the electron-positron system which is not accompanied by the processes of real electron-positron pair creation. In the case of the excited vacuum the wavefunctions of the vacuum electrons have both negative-frequency and positive-frequency components. The true meaning of the Klein mechanism of vacuum polarisation is that the 'bare' electron states are 'dressed' by the applied field and as a result the vacuum passes from the ground state to an excited one without creating real pairs. From the real pair creation criterion obtained it is inferred that no real pairs can be created by an arbitrary static field. It follows from the results presented that induced emission of the excited vacuum background is possible, i.e. transitions of the collective vacuum vibrations to the single quasi-particle states may occur. Deformation of the vacuum cannot be correctly taken into account within S-matrix scattering theory because in the latter this process is always interpeted as real pair creation.

1. Introduction

The main conclusion arrived at in this paper is that under the influence of an external field acting on the electron-positron system there takes place a restructuring (deformation) of its vacuum state not accompanied by the creation of real electron-positron pairs. The energy of the external field can be accumulated by vacuum vibrations, the vacuum going over to an excited state whose energy is greater than that of the ground vacuum state. The structure of the excited vacuum is defined by the value of the applied field strength. The excited vacuum background (by which we mean a medium consisting of virtual particles) may return to the ground state by emitting the surplus energy in the form of real photons and electron-positron pairs. Thus, one can formulate the fundamental problems of vacuum background pumping and stimulated emission of the excited vacuum state which are also of interest from the experimental viewpoint.

The deformation of the vacuum is closely connected with the restructuring of the energy spectrum of elementary excitations of a system which may be very large, especially in sufficiently strong external fields or in various resonance situations (Oleinik 1971, Belousov and Oleinik 1979). The energy of the external field goes into redistribution of the vacuum charges, accompanied by realignment of the energy spectrum, and it may be borrowed from the field not only in the form of an integral number of quanta

† The main ideas of the present paper were briefly outlined in Oleinik and Belousov (1983) and Oleinik (1983, 1984).

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but also continuously, in proportion to the intensity of the field. The energy spectrum deformation caused by virtual processes of 'dressing' the 'bare' states is responsible for many interesting physical effects (Oleinik 1971).

The results of this paper follow necessarily from comparing the Klein mechanism of vacuum polarisation in an external field with the mechanism of real pair creation predicted by Oleinik and Belousov (1983) and Oleinik and Arepjev (1984b). The latter mechanism is due to the appearance of quasi-stationary elementary excitations of the electron-positron system. It cannot be described within the S-matrix theory of quantum transitions.

A study of tunnel transitions and vacuum polarisation in an external field and also of the unitarity condition for the S operator and energy balance equations for quantised fields made in this paper reveals that S-matrix theory does not always describe correctly the processes of electron-positron pair creation and of emission and absorption of photons in the external field. This is due to the fact that in conventional scattering theory any deformation of the vacuum background is interpreted as the creation or absorption of real observable particles, whereas in actual fact the vacuum may be caused by an applied field to pass into such an excited state in which no real particles are present.

The result of the time evolution of the initial state $|n\rangle$ is usually written in the form of the following superposition:

$$S|n\rangle = \sum_m |m\rangle\langle m|S|n\rangle \tag{1}$$

where the matrix element $\langle m|S|n\rangle$ is interpreted as the probability amplitude for transition of a system from the state $|n\rangle$ to the state $|m\rangle$. According to the results obtained, in some cases the expansion (1) has a purely formal character and its coefficients $\langle m|S|n\rangle$ cannot be interpreted as the probability amplitudes for real quantum processes. The main point is that the ket-vectors $|m\rangle$ in (1) describe not only the real particles but also the virtual particles making up the vacuum background of quantised fields. It should be emphasised that the vacuum deformation caused by an applied field need not be accompanied by real physical processes.

In view of the existence of the vacuum background, which is in a sense similar in its physical properties to an elastic medium, the conventional interpretation of the S matrix is not always in agreement with the correspondence principle of quantum mechanics. In this paper a simple model is given in which S-matrix theory predicts the possibility of emission of a photon by an electron whereas, by the correspondence principle, the emission or absorption processes of real photons by electrons cannot take place.

In § 2 attention is drawn to the existence of two types of tunnelling transitions occurring under the action of an external field. This is necessary for a more complete understanding of the true nature of the Klein mechanism of vacuum polarisation. In § 3 the interrelationship is established between the mechanism of real electron-positron pair production predicted in Oleinik and Belousov (1983) and the Klein mechanism of the vacuum polarisation. Here the conclusion is drawn that the vacuum of quantised fields may be in an excited state and the criterion for real pair production in an external field is derived. The unitarity condition for the S matrix is examined in § 4. In § 5 an analysis is made of the energy balance equations for the interacting fields and of the conditions under which the transition probability concept may be defined. The main conclusions arrived at in this paper are summarised in § 6.

2. Two types of tunnel transitions

According to Blokhintsev (1961) and Oleinik and Belousov (1983) there are two types of electron tunnelling transitions through a potential barrier which differ considerably from one another in their physical characteristics: transitions of free electrons and of electrons localised in a potential well.

First, we consider the tunnelling transitions of free electrons moving towards the potential barrier from infinity (from now on they will be referred to as transitions of the first type). Let j_0 be an incident flux of electrons with energy E ($E < V_0$, where V_0 is the height of the one-dimensional square potential barrier). As the electron states with $E < V_0$ are doubly degenerate, one can always construct such a state which gives the non-vanishing incident flux ($j_0 \neq 0$). The flux j in the electron wave penetrating the barrier is defined by the transmission coefficient D: $j = Dj_0$. This type of tunnelling is a stationary process which is characterised by the absence of any sources or sinks of the current density of particles $\boldsymbol{j} \equiv \boldsymbol{j}(\boldsymbol{r}, t)$: $\operatorname{div} \boldsymbol{j} = 0$.

Now we turn to tunnelling transitions of the second type—penetration of a barrier by electrons initially localised in the potential well region (see the region $(-L, 0)$ in figure 1(a)). In this case the electron states with energy E ($E < V_0$) are non-degenerate. As the quantities j_0 and j are both equal to zero, the transmission coefficient D makes no sense. The electron tunnelling transitions out of the potential well cannot be described within the consistent stationary quantum mechanical theory.

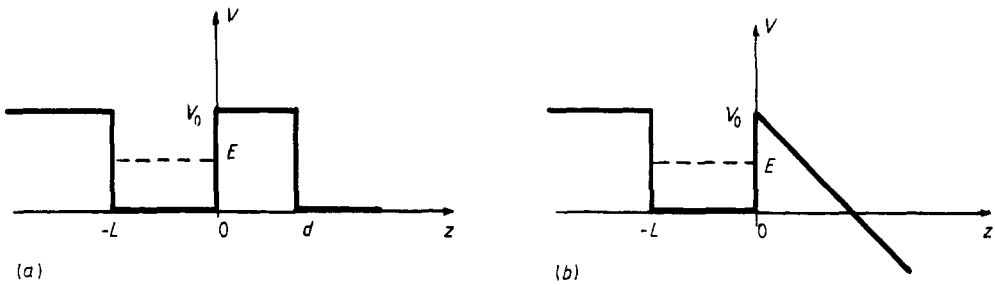

Figure 1. The potential well and the barrier.

In order to explain the second type of tunnelling the outgoing-wave boundary condition is imposed on the solution ψ of the Schrödinger equation for stationary states. This condition consists of the requirement that there be only outgoing waves far away from the barrier. With the potential energy curve depicted in figure 1(a), the outgoing-wave boundary condition is of the form

$$\psi = \exp(ikz) \qquad \text{at } z > d \qquad (k \equiv (2mE)^{1/2}).$$

The Schrödinger equation has a solution satisfying the above condition only at imaginary values of energy: $E = E' - i\Gamma$ ($E' \equiv \operatorname{Re} E, \Gamma > 0$), the solution being exponentially divergent at $z \to +\infty$. For this reason the stationary theory of the second type of tunnelling is not a correct quantum mechanical theory.

In view of the non-stationary character of the second type of tunnelling it should be described in terms of the wavefunction satisfying the time-dependent Schrödinger equation and an initial condition. Such an approach was developed by Drukarjev

(1951) when considering a particle transfer out of the potential well in the short-range potential case. In its application to the problem of a particle tunnelling out of the well under the influence of an electric field $\mathscr{E}$, this approach is as follows (Oleinik and Belousov 1983, Oleinik and Arepjev 1984a). Assume that prior to the moment $t=0$ a particle was located in a well and described by the wavefunction $\varphi_E(z, t)$ with energy E. Then at $t=0$ the electric field $\mathscr{E}$ is switched on in the half-plane $z>0$ (see figure 1(*b*)). The tunnel current density of electrons is given by

$$j_E(z, t)=\frac{\mathrm{i}}{2m}\psi_E(z, t)\overleftrightarrow{\partial}_z\psi_E^*(z, t)$$

where $\psi_E(z, t)$ is the solution of the time-dependent Schrödinger equation in the potential well and in the field $\mathscr{E}$ satisfying the initial condition $\psi_E(z, 0)=\varphi_E(z, 0)$. Within a certain time interval and for $t-az^{1/2}>0$, $z>0$ the smoothly varying time component of the current $j_E(z, t)$ is of the form

$$j_E(z, t)=\sum_n c_n\Gamma_n \exp[-2\Gamma_n(t-az^{1/2})] \tag{2}$$

where a ($a>0$) and c_n are constant coefficients, Γ_n is the width of a quasi-stationary state formed in the well and the symbol Σ_n means the sum over all quasi-stationary states. As is seen from (2), the second type of tunnelling is a decay of the bound stationary state of an electron under the influence of an electric field, the magnitude of the tunnel current being defined by the width Γ_n of quasi-stationary states. The particle flux emerging from the well of volume V is given by $\int_V \operatorname{div} \boldsymbol{j}\, d\boldsymbol{r}$. Therefore, the inequality $\operatorname{div} \boldsymbol{j}>0$ should be regarded as the necessary condition for the second type of tunnelling to occur.

It is clear from what has been said above that the first type of tunnelling is merely a free particle travelling from one spatial region to another one separated by a potential barrier. The second type of tunnelling is a different kind of process: it is a particle escaping from a certain region when there is no particle flux entering it.

3. Vacuum polarisation and pair creation

The conclusion above that there exist two different types of tunnelling is especially important when investigating the interband transitions (Klein 1929, Sauter 1931, Zener 1934, Schwinger 1951). In accordance with conventional theory the electron–positron pair creation in an electric field is a tunnelling penetration of an electron from the completely filled lower band (from the Dirac sea) into the upper one through the forbidden range. When considering the pair creation problem from the qualitative point of view, this process is usually described in terms of the transmission coefficient D evaluated for the negative-energy electron, with the forbidden band regarded as a potential barrier. In other words, the pair creation in an electric field is assumed to be the first type of tunnelling. The production of pairs in an electric field cannot be, however, a tunnel transition of the first type because in the lower band there is no flux of free particles falling on the potential barrier (on the forbidden band). Due to the fact that all the states in the lower band are filled by negative-energy electrons in accordance with the Pauli exclusion principle, the lower band is analogous to a potential well. Thus, for the problem in question the transmission coefficient D has no bearing on pair creation.

From the analogy indicated above one may conclude that there should exist a mechanism of real pair production analogous to the tunnelling of a particle out of a potential well. Such a conclusion drawn in Oleinik (1981) is confirmed by examining the exact solution of the Dirac equation in the square potential well and electric field (Oleinik and Arepjev 1984b). This type of pair production is due to the appearance of elementary excitations with the non-zero energy level width Γ and cannot therefore be described within S-matrix scattering theory. Indeed, in the latter the initial condition is formulated in the infinitely remote past (at $t_0 \to -\infty$) and scattered particles are registered in the infinitely remote future (at $t \to +\infty$). As was pointed out by Heitler (1954), such a statement of the scattering problem makes no sense for the systems with $\Gamma \neq 0$. It should be noted that passing to the limit mentioned above ($t_0 \to -\infty$, $t \to +\infty$) is essential in S-matrix theory.

To answer the question of the true meaning of the vacuum polarisation mechanism investigated by Klein (1929) for the first time (it will henceforth be referred to as the Klein mechanism), we consider the electron–positron field interacting with an external field $A_{\text{ext}}(x)$ switched on at $t = t_0$. The field operator may be written in the form

$$\Psi(x) = \sum_n (a_n \psi_n^{(+)}(x) + b_n^+ \psi_n^{(-)}(x)) \tag{3}$$

where $\psi_n^{(\pm)}(x)$ is the solution of the Dirac equation in the field $A_{\text{ext}}(x)$ satisfying the initial condition: $\psi_n^{(\pm)}(x) = \varphi_n^{(\pm)}(x)$ at $t \leqslant t_0$; $\varphi_n^{(+)}(x)(\varphi_n^{(-)}(x))$ is the wavefunction of a free electron (positron) with the quantum numbers n; a_n and b_n^+ are the second quantisation operators. The mean value of the 4-current density

$$j_\mu(x) = \tfrac{1}{2}[\bar{\Psi}(x)\gamma_\mu\Psi(x) - \Psi(x)\gamma_\mu^T\bar{\Psi}(x)] \tag{4}$$

in the vacuum state $|0\rangle$ is given by

$$\begin{aligned} \langle 0|j_\mu(x)|0\rangle &= \sum_n \bar{\psi}_n^{(-)}(x)\gamma_\mu\psi_n^{(-)}(x) - \sum_n \bar{\varphi}_n^{(-)}(x)\gamma_\mu\varphi_n^{(-)}(x) \\ &\equiv (\rho_{\text{vac}}(x), \boldsymbol{j}_{\text{vac}}(x)) = j_{\text{vac},\mu}(x). \end{aligned} \tag{5}$$

The second term on the right-hand side of (5) is a constant quantity not depending on $A_{\text{ext}}(x)$. Therefore it may be neglected.

Recall that the electron–positron pair creation is a quantum process in which a particle goes over from the negative-energy state to the positive-energy one. A hole appearing in the range of negative-energy states is interpreted as a positron. The creation of a positron at a certain point $\boldsymbol{r}$ may take place only if this point is a source of the vacuum current $\boldsymbol{j}_{\text{vac}}(x)$. In other words, the condition for creating an electron–positron pair in an external field should be expressed by the inequality

$$\operatorname{div} \boldsymbol{j}_{\text{vac}}(x) \neq 0. \tag{6}$$

By integrating the continuity equation for the 4-vector (5) over the volume V, we arrive at the relation

$$-\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_{\text{vac}}(x)\,\mathrm{d}\boldsymbol{r} = \int_V \operatorname{div} \boldsymbol{j}_{\text{vac}}(x)\,\mathrm{d}\boldsymbol{r} \tag{7}$$

whose physical meaning is obvious. The left-hand side of (7) represents the change in the number of particles (of electrons and positrons) in the volume V per unit time and the right-hand side is reduced to the flux of particles emerging from the volume V. Both processes, the tunnelling transitions of electrons out of the well in an electric

field and the electron–positron pair creation caused by an external field, are thus entirely analogous.

Obviously, from the inequality

$$j_{\text{vac}}(x) \neq 0 \tag{8}$$

which is supposed to be valid in a volume V, it does not follow that pairs are created in this volume. Really, if $\operatorname{div} j_{\text{vac}}(x) = 0$ in the volume V, the inequality (8) is only indicative of the fact that in this range the Dirac sea (i.e. the set of all the states described by the $\psi_n^{(-)}(x)$ functions) is shifted as a whole without vacancies forming in it. Such a situation takes place in a homogeneous electric field with the intensity $\mathscr{E}(t)$ arbitrarily depending on time. If the field is described by the 3-potential $\mathscr{A}(t) = -\int_0^t \mathscr{E}(t')\,\mathrm{d}t'$, the wavefunction $\psi_n^{(\pm)}(x) = \psi_{p\sigma}^{(\pm)}(x)$ is of the form

$$\psi_{p\sigma}^{(\pm)}(x) = R_{p\sigma}^{(\pm)}(t)\exp(\mathrm{i}\boldsymbol{p}\boldsymbol{r}) \tag{9}$$

where $R_{p\sigma}^{(\pm)}(t)$ is a time-dependent bispinor, $\boldsymbol{p}$ and σ are the momentum and the spin variable, respectively. By virtue of (5) and (9) the vacuum current $j_{\text{vac}}(x)$ is non-vanishing but it does not depend on $\boldsymbol{r}$ and thus pairs are not created in the field $\mathscr{E}(t)$.

According to the criterion for pair production indicated above, an arbitrary static external field cannot produce a pair. Indeed, in such a field the quantity $\rho_{\text{vac}}(x)$ is time independent; therefore, by virtue of the continuity equation for the 4-vector (5) $\operatorname{div} j_{\text{vac}}(x) = 0$.

Let us now investigate the pair production problem in the field $\mathscr{E}(t)$ from the viewpoint of S-matrix theory. Suppose for simplicity that the external field $\mathscr{E}(t)$ is distinct from zero only within the time interval (t_0, t_1). At $t > t_1$ the wavefunction $\psi_n^{(-)}(x)$ introduced above can be written in the form

$$\psi_n^{(-)}(x) = \sum_m [\alpha_{nm}\varphi_m^{(+)}(x) + \beta_{nm}\varphi_m^{(-)}(x)] \tag{10}$$

where α_{nm} and β_{nm} are constant coefficients. The appearance of the positive-energy components on the right-hand side of (10) is usually interpreted as real electron–positron pair creation, the quantity

$$\alpha_{nm} = (\varphi_m^{(+)}, \psi_n^{(-)})_t$$

being taken as the probability amplitude for pair creation. Here $(\varphi_m^{(+)}, \psi_n^{(-)})_t$ is the scalar product of the wavefunctions $\varphi_m^{(+)}(x)$ and $\psi_n^{(-)}(x)$. Thus, from the S-matrix theory viewpoint the pair production criterion is expressed by the inequality

$$\alpha_{nm} \neq 0 \tag{11}$$

which means that the positive-energy components $\varphi_m^{(+)}(x)$ are present in the expansion of $\psi_n^{(-)}(x)$ in terms of unperturbed wavefunctions. The number of pairs created in the external field is believed to be given by

$$N = \sum_{nm} |\alpha_{nm}|^2. \tag{12}$$

The quantity (12) is just the number of pairs produced by Klein mechanism. The condition (11) is far from being equivalent to (6). Indeed, in the field $\mathscr{E}(t)$ the inequality (11) is fulfilled though $\operatorname{div} j_{\text{vac}}(x) = 0$. As was explained above, the last equality means that there are no vacuum current sources in the field $\mathscr{E}(t)$, i.e. holes are not formed in the Dirac sea.

To find out the correct interpretation of the quantity N, let us introduce the subspaces $M_0^{(+)}$ and $M_0^{(-)}$ which are orthogonal to each and are formed by linear combinations of the functions $\varphi_n^{(+)}$ and $\varphi_n^{(-)}$, respectively. These subspaces describe the upper and lower continua. In the field $A_{ext}(x)$ these subspaces are distorted to go over to the $M^{(+)}(t)$ and $M^{(-)}(t)$ subspaces whose basis vectors are the functions $\psi_n^{(+)}(x)$ and $\psi_n^{(-)}(x)$, respectively.

The mean value of the operator $j_\mu(x)$ (4) in the vacuum state can be written as follows:

$$j_{vac,\mu}(x) = \tfrac{1}{2}\sum_n (\bar\psi_n^{(-)}(x)\gamma_\mu\psi_n^{(-)}(x) - \bar\psi_n^{(+)}(x)\gamma_\mu\psi_n^{(+)}(x)). \tag{13}$$

Making use of (5) and (13), we arrive at the equalities

$$\begin{aligned} &\mathcal{J}^{(-)}(x) = -\mathcal{J}^{(+)}(x), \\ &\mathcal{J}^{(\pm)}(x) = \tfrac{1}{2}\sum_n (\bar\psi_n^{(\pm)}(x)\boldsymbol{\gamma}\psi_n^{(\pm)}(x) - \bar\varphi_n^{(\pm)}(x)\boldsymbol{\gamma}\varphi_n^{(\pm)}(x)). \end{aligned} \tag{14}$$

Obviously, the quantities $\mathcal{J}^{(\pm)}(x)$ may be interpreted as the currents induced by an applied field in the subspaces $M^{(\pm)}(t)$. According to (14) the appearance of a current density source in the subspace $M^{(-)}(t)$ is necessarily accompanied by a current density sink in the subspace $M^{(+)}(t)$ occurring: $\operatorname{div}\mathcal{J}^{(-)}(x) = -\operatorname{div}\mathcal{J}^{(+)}(x)$. The total vacuum current is defined by

$$\boldsymbol{j}_{vac}(x) = \mathcal{J}^{(-)}(x) - \mathcal{J}^{(+)}(x) = 2\mathcal{J}^{(-)}(x). \tag{15}$$

It is natural to connect the real pair production with the appearance of sources of the vectors $\mathcal{J}^{(-)}(x)$ and $-\mathcal{J}^{(+)}(x)$. If the transition $M_0^{(-)} \to M^{(-)}(t)$ is merely a deformation of the lower continuum as a whole and is not accompanied by the sources of the vector $\mathcal{J}^{(-)}(x)$ appearing in it, the real electron-positron pairs are of course not produced. Obviously, the quantity N (12) represents the number of states in the intersection of the subspaces $M^{(-)}(t)$ and $M_0^{(+)}$ and may be considered as a measure of the vacuum polarisation in an external field. But this type of vacuum polarisation is not necessarily accompanied by real pair creation. If the condition (6) is not fulfilled, the 'positrons' whose number is given by (12) are frozen in the Dirac sea. They are a part of the Dirac sea and, therefore, cannot be experimentally registered as real physical particles.

Making use of the known expression for the momentum density operator of the electron-positron field:

$$\boldsymbol{T}(x) = \frac{i}{4}\left(\bar\Psi(x)\boldsymbol{\gamma}\frac{\partial}{\partial t}\Psi(x) - \frac{\partial\Psi(x)}{\partial t}\boldsymbol{\gamma}^T\bar\Psi(x) - \frac{\partial\bar\Psi(x)}{\partial t}\boldsymbol{\gamma}\Psi(x) + \Psi(x)\boldsymbol{\gamma}^T\frac{\partial\bar\Psi(x)}{\partial t}\right) \tag{16}$$

one can easily show that the subspaces in question may be characterised by the momenta $\boldsymbol{T}^{(\pm)}(x) = \boldsymbol{T}^{(\pm)}$, where

$$\boldsymbol{T}^{(\pm)} = \frac{i}{4}\sum_n\left(\bar\psi_n^{(\pm)}\boldsymbol{\gamma}\frac{\partial\psi_n^{(\pm)}}{\partial t} - \frac{\partial\bar\psi_n^{(\pm)}}{\partial t}\boldsymbol{\gamma}\psi_n^{(\pm)}\right) - \frac{i}{4}\sum_n\left(\bar\varphi_n^{(\pm)}\boldsymbol{\gamma}\frac{\partial\varphi_n^{(\pm)}}{\partial t} - \frac{\partial\bar\varphi_n^{(\pm)}}{\partial t}\boldsymbol{\gamma}\varphi_n^{(\pm)}\right)$$

$$-\frac{e}{4}\sum_n(\bar\varphi_n^{(\pm)}\boldsymbol{\gamma}\gamma_0\hat A_{ext}\varphi_n^{(\pm)} + \bar\varphi_n^{(\pm)}\hat A_{ext}\gamma_0\boldsymbol{\gamma}\varphi_n^{(\pm)}).$$

The following equalities analogous to (14) and (15) are fulfilled:

$$\boldsymbol{T}^{(-)} = -\boldsymbol{T}^{(+)} \qquad \langle 0|\boldsymbol{T}|0\rangle = 2\boldsymbol{T}^{(-)}.$$

To elucidate the main conclusions drawn above, let us turn to the field operator $\Psi(x)$ (3) which describes the time evolution of free particles in the field A_{ext}. Making use of (3) and (4), we arrive at the relations

$$\langle 0|a_n j_\mu(x) a_n^+|0\rangle = \bar{\psi}_n^{(+)}(x)\gamma_\mu\psi_n^{(+)}(x) + \langle 0|j_\mu(x)|0\rangle$$
$$\langle 0|b_n j_\mu(x) b_n^+|0\rangle = \bar{\psi}_n^{(-)}(x)\gamma_\mu\psi_n^{(-)}(x) + \langle 0|j_\mu(x)|0\rangle$$

from which it follows that the quantity $\bar{\psi}_n^{(\pm)}(x)\gamma_\mu\psi_n^{(\pm)}(x)$ may be interpreted as the 4-current density of a particle in the field $A_{\text{ext}}(x)$. Therefore, the transition $\varphi_n^{(\pm)}(x) \to \psi_n^{(\pm)}(x)$ may be considered as a 'dressing' of a free state in an external field and the vacuum current $j_{\text{vac},\mu}(x)$ as the total current of the 'dressed' states which were described by the $\varphi_n^{(-)}(x)$ functions prior to the moment of the switching-on of the field $A_{\text{ext}}(x)$. Thus, the vacuum current $j_{\text{vac}}(x)$ is due to a 'dressing' of free states of which the lower continuum consists. The subspace $M^{(-)}(t)$ introduced above describes the Dirac sea of 'dressed' particles. Obviously, pair creation in an external field should be associated with the formation of a hole in the Dirac sea of 'dressed' rather than 'bare' states. The appearance of a hole leads to a change of the charge density $\rho_{\text{vac}}(x)$ of the Dirac sea. In other words, the condition for real pair creation within the time interval (t_1, t_2) should be expressed by the inequality

$$\partial\rho_{\text{vac}}(x)/\partial t \neq 0 \qquad \text{at } t \in (t_1, t_2).$$

By virtue of the continuity equation this inequality is equivalent to (6). We may conclude from what has been said that pair creation in an external field being predicted by S-matrix theory is a purely virtual process of 'dressing' the 'bare' states. To experimentally detect an electron–positron pair, one should spatially separate the electric charges forming the pair. If the relation $\operatorname{div} j_{\text{vac}}(x) = 0$ is fulfilled in a spatial region, this means that such a separation cannot be carried out and therefore there is, in principle, no way of registering an electron–positron pair in this region.

In the lowest order of the perturbation theory

$$(\varphi_n^{(+)}, \psi_m^{(-)})\Big|_{\substack{t\to+\infty \\ t_0\to-\infty}} = -\mathrm{i}\int_{-\infty}^{+\infty} \mathrm{d}x\, \bar{\varphi}_n^{(+)}(x) e\hat{A}_{\text{ext}}(x)\varphi_m^{(-)}(x).$$

Within S-matrix theory, the last quantity is interpreted as the amplitude for real pair creation in the field $A_{\text{ext}}(x)$. Pairs, however, are not created in the field $A_{\text{ext}} = -\mathcal{E}\omega^{-1}\sin\omega t$, which is shown by the equality $\operatorname{div} j_{\text{vac}}(x) = 0$ that is fulfilled in this case. From this we may conclude that it is possible for the vacuum to absorb the energy of an applied field with frequency ω (at $\omega > 2m$) without creating real pairs. Thus, electromagnetic energy may be accumulated by the vacuum vibrations. The electromagnetic field energy is expended in redistribution of the vacuum charges ($j_{\text{vac}}(x) \neq 0$) and is stored in the form of distortions of the vacuum background. Obviously, a reverse process can occur as well: transformation of the collective vacuum excitations into single quasi-particles.

The structure of the vacuum background deserves a more detailed discussion. The appearance of positive-energy components on the right-hand side of (10) is usually interpreted as real electron–positron pair creation. This interpretation was shown above to be incorrect. The inequality $\alpha_{nm} \neq 0$ only means that the applied external field deforms the Dirac sea and is absorbed by the vacuum vibrations. The subspace $M^{(-)}(t)$ describing the Dirac sea of the 'dressed' states at an instant t coincides at $t < t_0$ with the subspace $M_0^{(-)}$ of negative-energy states but its basis at $t > t_1$ is the set

of functions defined by the right-hand side of (10). The vacuum electrons at $t > t_1$ are in the non-stationary states containing both the positive-energy components and the negative-energy ones. Thus, the vacuum at $t > t_1$ is in an excited state which considerably differs from the ground state of the vacuum at $t < t_0$.

An analogous situation arises with the gravitational field interacting with scalar particles at the spherically symmetrical collapse (Hawking 1975). One can show that in the subspace $M^{(-)}(t)$ the scalar particle sources do not appear in this case. Thus, the vacuum background is merely deformed by the spacetime metric dependent on time and real particles are not created in the black hole field.

When investigating the quantum processes, the vacuum background is usually assumed to consist of negative-energy electrons. As is seen from the results obtained, this is not always the case because the vacuum can be in an excited state. The specific structure of the excited vacuum is defined by the coefficients α_{nm} and β_{nm} in (10) and depends upon the vacuum pumping by an applied field. This gives rise to the question: what is the structure of the vacuum in the real world? (On the vacuum invariance problem see Coleman (1967).)

4. Unitarity condition

Let us consider the unitarity condition for the S matrix

$$\sum_f \langle n|S^+|f\rangle\langle f|S|n\rangle = 1 \tag{17}$$

where $|n\rangle$ is an arbitrary ket-vector and Σ_f means the sum over a complete set of ket-vectors $|f\rangle$. Separate out in (17) the terms corresponding to elastic scattering processes, i.e. to processes for which the ket-vectors $|n\rangle$ and $|f\rangle$ describe the states with the same number of particles and may differ from one another only by the quantum numbers pertaining to particles. The ket-vectors relating to elastic and inelastic scattering will be denoted by $|f_0\rangle$ and $|f_r\rangle$ $(r = 1, 2, \ldots)$, respectively. The index r is used for numbering the different channels via which inelastic scattering may take place. Then the unitarity condition (17) can be represented as follows:

$$\sum_{r=0,1,\ldots} W_r^{(n)} = 1 \qquad W_r^{(n)} \equiv \sum_{f_r} |\langle f_r|S|n\rangle|^2. \tag{18}$$

The quantity $W_r^{(n)}$ is the total transition probability of a system from state $|n\rangle$ to states $|f\rangle$ corresponding to the r channel of scattering. The above interpretation of the quantities $W_r^{(n)}$ is correct, evidently, only if the following inequalities are fulfilled:

$$0 \leqslant W_r^{(n)} \leqslant 1 \qquad (r = 0, 1, \ldots). \tag{19}$$

For a system homogeneous in time the S-matrix elements are usually expressed in terms of the $\mathcal{T}$-matrix elements:

$$\langle f_r|S|n\rangle = \delta_{fn}\delta_{r0} + i\delta(E_f - E_n)\mathcal{T}_{fn}^{(r)} \tag{20}$$

where E_f and E_n are energies of the system in the states $|f_r\rangle$ and $|n\rangle$. The first term on the right-hand side of (20) corresponds to the absence of scattering and the second

one takes account of the law of energy conservation. Substituting (20) into (18), we obtain

$$\left[1+\delta(0)\left(\mathrm{i}(\mathscr{T}_{nn}^{(0)}-\mathscr{T}_{nn}^{*(0)})+\sum_{f_0}\delta(E_f-E_n)|\mathscr{T}_{fn}^{(0)}|^2\right)\right]+\delta(0)\sum_{r=1,2,\ldots}\sum_{f_r}\delta(E_f-E_n)|\mathscr{T}_{fn}^{(r)}|^2=1 \tag{21}$$

$$\begin{aligned} W_0^{(n)} &= 1+(T/2\pi)\left[\mathrm{i}(\mathscr{T}_{nn}^{(0)}-\mathscr{T}_{nn}^{*(0)})+\sum_{f_0}\delta(E_f-E_n)|\mathscr{T}_{fn}^{(0)}|^2\right] \\ W_r^{(n)} &= (T/2\pi)\sum_{f_r}\delta(E_f-E_n)|\mathscr{T}_{fn}^{(r)}|^2 \qquad (r=1,2,\ldots). \end{aligned} \tag{22}$$

In (22) we have replaced, as usual, the quantity $\delta(0)$ by $T/2\pi$, where T is interpreted as the interaction time ($T\to\infty$).

As is seen from (22), the conditions (19) can be fulfilled only if the following equalities are valid:

$$\begin{aligned} &\mathrm{i}(\mathscr{T}_{nn}^{(0)}-\mathscr{T}_{nn}^{*(0)})+\sum_{f_0}\delta(E_f-E_n)|\mathscr{T}_{fn}^{(0)}|^2=0 \\ &\mathscr{T}_{fn}^{(r)}=0 \qquad (r=1,2,\ldots). \end{aligned} \tag{23}$$

Consequently, for a system homogeneous in time the concept of transition probability may be introduced solely when inelastic scattering does not occur. If inelastic scattering processes are allowed, the relations (18)-(20) become incompatible.

Consider now quantum transitions of a system from the state $|n\rangle$ to the state $|f\rangle$ belonging to a set M, with the state $|n\rangle$ not belonging to M. By virtue of (20)

$$|\langle f_r|S|n\rangle|^2=(T/2\pi)\delta(E_f-E_n)|\mathscr{T}_{fn}^{(r)}|^2 \qquad T\to\infty. \tag{24}$$

The total probability of the transition $|n\rangle\to|f\rangle$, with $|f\rangle$ belonging to M, in unit time is given by

$$W_n=\frac{1}{T}\sum_r\sum_{f_r}{}'|\langle f_r|S|n\rangle|^2 \qquad \sum_{f_r}{}'\equiv\sum_{f_r(|f_r\rangle\in M)}. \tag{25}$$

From (25) and (18) it follows that $W_n<T^{-1}$. As $T\to\infty$, we have

$$W_n=0. \tag{26}$$

The last equality holds for arbitrary set M and arbitrary initial state $|n\rangle$ not belonging to M. The equality (26) seems to be an indication that the S matrix relating to a system homogeneous in time has no physical information.

The results obtained above are exact, in the sense that perturbation theory has not been used in deriving them. They are of a rather general nature and hold true in particular for a system placed in an external field periodically varying in time (a special case of such a system has been considered in Oleinik and Belousov (1983)).

To illustrate the above conclusions we consider, by applying perturbation theory, the quantised fields interacting with a homogeneous magnetic field of intensity $\boldsymbol{H}$. For definiteness, the magnetic field will be described by the 3-potential $\boldsymbol{A}=(-yH,0,0)$, $H=$ constant. In this case the electron states can be characterised by the quantum numbers p_x, p_z, n and σ (p_x and p_z are the momentum components, n and σ are the number of the Landau level and the spin index, respectively). As the states $|n\rangle$ and $|f_r\rangle$ $(r=0,1)$ we take the following ones:

$$|n\rangle=a^+_{p_x p_z n\sigma}|0\rangle \qquad |f_0\rangle=a^+_{p_x' p_z' n'\sigma'}|0\rangle \qquad |f_1\rangle=a^+_{p_x' p_z' n'\sigma'}c^+_{k\lambda}|0\rangle \tag{27}$$

where $a^+_{p_x p_z n\sigma}$ and $c^+_{k\lambda}$ are the creation operators for the electron and photon, respectively. We have

$$\langle f_0|S|n\rangle = \delta_{p_x p_x'}\delta_{p_z p_z'}\delta_{nn'}\delta_{\sigma\sigma'} + i\delta(\varepsilon_{p_z n'\sigma'} - \varepsilon_{p_z n\sigma})\delta_{p_x p_x'}\delta_{p_z p_z'}\mathcal{T}_{p_x p_z; n'\sigma' n\sigma}$$
$$\varepsilon_{p_z n\sigma} = [m^2 + p_z^2 + eH(2n + 1 - \sigma)]^{1/2}. \tag{28}$$

Retaining everywhere only terms of the order of α (α is the coupling constant describing the interaction of the electron with the quantised electromagnetic field, the electron's interaction with the magnetic field H being taken precisely into account) we obtain:

$$W_0^{(n)} = 1 + (T/2\pi)i(\mathcal{T}_{p_x p_z; n\sigma n\sigma} - \mathcal{T}^*_{p_x p_z; n\sigma n\sigma}) = 1 - W_1^{(n)}$$
$$W_1^{(n)} = \sum_{f_1} |\langle f_1|S|n\rangle|^2 \qquad \mathcal{T}_{p_x p_z; n'\sigma' n\sigma} \sim \alpha. \tag{29}$$

The quantity $W_1^{(n)}$ represents the total probability for emission of a photon by an electron in a magnetic field. As this quantity is proportional to T ($T \to \infty$), up to terms of first order in α we have $W_0^{(n)} \to -\infty$, $W_1^{(n)} \to +\infty$. Thus, the unitarity condition is formally fulfilled, but it does not allow one to introduce the concept of transition probability. The above conclusions may be checked by direct calculation to remain valid when computing the transition probability by the perturbation theory method up to terms of any order in α.

One may be puzzled by the fact that by definition (18) the quantity $W_0^{(n)}$ must be non-negative, but from (29) it follows that $W_0^{(n)} < 0$ up to terms of the first order in α. Here we have made, however, no mistake. With the accuracy indicated above the S-matrix element of elastic scattering is given by (28). It is obvious that the quantity $W_0^{(n)}$ contains the α^2 terms in addition. But they should be neglected if we confine ourselves to the terms of first order in α. Omitting the α^2 terms we obtain just the inequality mentioned above: $W_0^{(n)} = 1 - W_1^{(n)} < 0$. If the calculation is carried out up to α^2 terms, the S-matrix elements $\langle f_r|S|n\rangle$ should be computed with the same accuracy. But then the quantities $W_0^{(n)}$ will contain the α^3 and α^4 terms. Omitting these terms, we again arrive at the inequality $W_0^{(n)} < 0$ and so on.

As the state $|n\rangle$ we have used in (17), (18) and (29) a stationary state. All the results obtained remain valid also in the case when the stationary state $|n\rangle$ is replaced by the wave packet $|\phi\rangle = \Sigma_n c_n|n\rangle$, where c_n are arbitrary constants. Without losing the generality, as a complete set of ket-vectors $\{|f\rangle\}$ in (17), the stationary states may be used. Making use of the model above with the homogeneous magnetic field, one can easily show that $W_1^{(\phi)} \equiv \Sigma_{f_1} |\langle f_1|S|\phi\rangle|^2$ is proportional to T. The results obtained may be confirmed by a large number of specific models, for example the Čerenkov–Vavilov emission in a medium with the electric permittivity $\varepsilon > 1$, the emission of a photon by an electron in the external field $\boldsymbol{A} = \boldsymbol{a}\cos\omega t$ ($\boldsymbol{a}$ = constant) and others. In all these models the quantity $W_r^{(\phi)}$ ($r = 1, 2, \ldots$) is proportional to T, no matter whether the single-particle state $|\phi\rangle$ is a wave packet state or a stationary one.

According to (22) $W_r^{(n)}$ ($r = 1, 2, \ldots$) is proportional to T. The quantity T owes its appearance to the $\delta(E_f - E_n)$ function in the S-matrix element (20). More precisely, it results from the representation $\delta(0) = T/2\pi$ which is true only when $T \to \infty$. On the other hand, as is well known, the formulae (22) for transition probability are valid at not very large values of T (Dirac 1958). Otherwise, the probability may prove to be greater than unity.

At first glance, it is the appearance of the δ function in (20) that is the most alluring feature of the transition amplitude. The formula (20) may be checked by expanding

the S matrix in coupling constant. However, as was shown by Abakarov and Oleinik (1973), the proportionality itself of the transition amplitude to the δ function dependent on the energy difference is indicative that the perturbation theory expansion for the transition amplitude has zero radius of convergence.

The physical meaning of the results obtained is that any real physical system cannot be, in principle, homogeneous in time. Indeed, when describing the behaviour of a physical system it is necessary to take into account both the vacuum fluctuations and the classical fields related to the means of observation. In such an approach, the states of 'dressed' particles become quasi-stationary, i.e. the wavefunctions of elementary excitations of interacting fields are exponentially damped in time, and consequently the system ceases to be homogeneous in time (Oleinik and Belousov 1983). Naturally, because of the presence of damping the transition amplitudes cannot have the structure indicated on the right-hand side of (20). The equality (26) seems to mean that, while studying the quantum processes, it is necessary to take into consideration the non-zero width of energy levels of elementary excitations.

5. Energy balance equations

Let us consider the set of quantised electromagnetic $A_\mu(x) = (A_0(x), \boldsymbol{A}(x))$ and electron-positron $\Psi(x)$ fields interacting with an applied external field. As the external field we take the homogeneous electric field $\boldsymbol{A}_{\text{ext}}(t)$ which arbitrarily depends on time and is switched on at the instant $t = t_0$. For simplicity, the Coulomb gauge for the 4-potential of the electromagnetic field is used and the Coulomb field is neglected: $\operatorname{div} \boldsymbol{A}(x) = A_0(x) = 0$. The results obtained can easily be generalised to a more realistic model in which all the components of the 4-potential $A_\mu(x)$ are distinct from zero.

Making use of the Maxwell and Dirac equations for the field operators, one can readily derive the operator equation

$$\frac{\partial}{\partial t} W(x) + \operatorname{div} \boldsymbol{P}(x) + \boldsymbol{j}(x)\boldsymbol{E}(x) = 0 \tag{30}$$

which coincides in its form with the Poynting equation of classical electrodynamics. Here $W(x)$ and $\boldsymbol{P}(x)$ are the operators of the energy density and of the Poynting vector for the quantised electromagnetic field:

$$W(x) = \tfrac{1}{2}(\boldsymbol{E}^2(x) + \boldsymbol{H}^2(x)) \qquad \boldsymbol{P}(x) = [\boldsymbol{E}(x)\boldsymbol{H}(x)]$$

$$\boldsymbol{E}(x) = -\frac{\partial}{\partial t}\boldsymbol{A}(x) \qquad \boldsymbol{H}(x) = \operatorname{rot} \boldsymbol{A}(x).$$

The operators above have all been written in the Heisenberg picture. The operators $W(x)$, $\boldsymbol{P}(x)$ and others are functionals dependent on the 4-potential of the external field $A_{\text{ext}}(x) = (0, \boldsymbol{A}_{\text{ext}}(t))$.

For convenience, we go over to the Furry picture with the external field included in the unperturbed Hamiltonian. The field operators are defined by

$$A_\mu(x) = S_t^+ A_\mu^{(0)}(x) S_t \qquad \Psi(x) = S_t^+ \Psi^{(1)}(x) S_t \tag{31}$$

$$S_t = \hat{T} \exp\left(-\mathrm{i} \sum_\mu g^{\mu\mu} \int_{t_0}^{t} \mathrm{d}t_1 \int \mathrm{d}\boldsymbol{r}_1 \, j_\mu^{(1)}(x_1) A_\mu^{(0)}(x_1) \right)$$

where the superscripts '0' and '1' pertain to the operators in the interaction picture and in the Furry picture, respectively.

From (30) we get

$$-\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}\boldsymbol{r}\langle n|W(x)|n\rangle = \int_V \mathrm{d}\boldsymbol{r}\,\mathrm{div}\langle n|\boldsymbol{P}(x)|n\rangle + \int_V \mathrm{d}\boldsymbol{r}\langle n|\boldsymbol{j}(x)\boldsymbol{E}(x)|n\rangle \tag{32}$$

where V is a volume and $|n\rangle$ is an arbitrary state. Using the correspondence principle with the classical theory, the relation (32) may be interpreted as follows. The change in the electromagnetic energy in the state $|n\rangle$ per unit time in the volume V is equal to the sum of the energy flux of radiation and of the work done by the field $\boldsymbol{E}(x)$ on the electric current $\boldsymbol{j}(x)$ in unit time. It follows from the correspondence principle that in quantum electrodynamics equation (32) should play the same part which is played by the Poynting equation in classical electrodynamics.

Let us consider the matrix element of the time evolution operator S_t which corresponds to emission of a photon by an electron:

$$M_{n\to mk\lambda}(t) = \mathrm{i}e\sum_{\mu} g^{\mu\mu}\int_{t_0}^{t}\mathrm{d}t_1\int \mathrm{d}\boldsymbol{r}_1\,\bar{\psi}_m^{(+)}(x_1)\gamma_\mu \mathscr{A}_{k\mu}^{*(\lambda)}(x_1)\psi_n^{(+)}(x_1)g^{\lambda\lambda} \tag{33}$$

where $\mathscr{A}_{k\mu}^{(\lambda)}(x) = (2\omega V)^{-1/2}e_\mu^{(\lambda)}\exp(-\mathrm{i}kx)$ is the photon wavefunction, $k = (\omega, \boldsymbol{k})$ and $e_\mu^{(\lambda)}$ are the 4-momentum and the photon polarisation 4-vector, respectively V is the normalisation volume and $\psi_n^{(+)}(x)$ is the electron wavefunction in the field $A_{\mathrm{ext}}(x)$. The quantity $M_{n\to mk\lambda}(t)$ is known to be distinct from zero at $t\to+\infty$, $t_0\to-\infty$ if only $A_{\mathrm{ext}}(x)\neq 0$. This is usually interpreted in such a manner: in an external field (for instance in the field $\boldsymbol{A}_{\mathrm{ext}}(t) = \boldsymbol{a}\cos\omega t$, $\boldsymbol{a}$ = constant) the emission of a photon by an electron can occur, the quantity $M_{n\to mk\lambda}$ being the transition amplitude for the process under study in the lowest order of perturbation theory. Making use of this interpretation we arrive at the following formulae for the total probability $p_n(t)$ of emission of a photon by an electron and for the change in the radiated energy in unit time (in the lowest order of perturbation theory in the interaction with the quantised electromagnetic field):

$$p_n(t) = \sum_{k\lambda}\sum_{m}|M_{n\to mk\lambda}(t)|^2 \tag{34}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}w_n(t) = \frac{\mathrm{d}}{\mathrm{d}t}\sum_{k\lambda}\omega_k\sum_{m}|M_{n\to mk\lambda}(t)|^2. \tag{35}$$

It may readily be shown that the quantity $(\mathrm{d}/\mathrm{d}t)w_n(t)$ defined by the expression on the right-hand side of (35) can be written in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}w_n(t) = \frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}\boldsymbol{r}\langle 0|a_n W(x)a_n^+|0\rangle. \tag{36}$$

According to (36) the quantity $(\mathrm{d}/\mathrm{d}t)w_n(t)$ represents the change of the electromagnetic energy in the single-electron state $a_n^+|0\rangle$ in unit time in the whole space. On the other hand, by virtue of the relation (35) which results from S-matrix scattering theory, the quantity $(\mathrm{d}/\mathrm{d}t)w_n(t)$ should be interpreted as the total energy emitted by an electron in the form of real electromagnetic quanta. Such an interpretation disagrees, however, with the correspondence principle. Indeed, by the correspondence principle the radiated energy flux in the state $|n\rangle$ is defined by $\int_V \mathrm{d}\boldsymbol{r}\,\mathrm{div}\langle n|\boldsymbol{P}(x)|n\rangle$. But from the space

homogeneity of the system in question it follows that the quantity $\boldsymbol{P}_n(x) \equiv \langle 0|a_n \boldsymbol{P}(x) a_n^+|0\rangle$ is independent of $\boldsymbol{r}$, i.e.

$$\operatorname{div} \boldsymbol{P}_n(x) = 0. \tag{37}$$

Consequently, in the state $a_n^+|0\rangle$ there are no sources and sinks of the Poynting vector. Therefore, in this state the processes of emission and absorption of real photons by electrons do not occur. From (32) and (37) it follows that in the single-electron state the energy of the electromagnetic field is expended only in the work done by the field $\boldsymbol{E}(x)$ on the electric current $\boldsymbol{j}(x)$.

We now consider in more detail what this work represents. It is easily seen that $\boldsymbol{P}_n(x) \not\equiv 0$ *and* $\boldsymbol{P}_n(x)|_{t=t_0} = 0$. This means that the virtual photons of which the free electromagnetic field $A_\mu^{(0)}(x)$ consists begin to be excited when a perturbation is switched on. Naturally, at the same time the real electron corresponding to the state n and virtual particles related to the field $\Psi^{(0)}(x)$ are set in motion as well. It is the work done by the field $\boldsymbol{E}(x)$ both on the real electron in the state n and on the virtual electronic charges that is given by the quantity $\int_V \mathrm{d}\boldsymbol{r}\langle 0|a_n \boldsymbol{j}(x)\boldsymbol{E}(x)a_n^+|0\rangle$.

The results obtained may be interpreted in the following way. The quantised fields $A_\mu^{(0)}(x)$ and $\Psi^{(0)}(x)$ corresponding to non-interacting particles describe a medium which we call the vacuum background. When a perturbation is switched on, both the real particles and the vacuum background are set in motion. The energy is constantly transferred from the photon subsystem to the electron-positron one and back again. As a result of this process, the vacuum background is distorted. The work done by the electromagnetic field on real charges and on the vacuum background is described for the second term on the right-hand side of (32). As is seen from (37), when a single-electron state is taken as the initial state of the quantised fields being perturbed by the external field $\boldsymbol{A}_{\mathrm{ext}}(t)$, the displacement of the real electron and the vacuum background deformation are the only result of electron-photon interaction.

By (32) the processes of real photon emission are approximately described by S-matrix theory only in the case when the following condition is fulfilled:

$$\left|\int_V \mathrm{d}\boldsymbol{r}\langle n|\boldsymbol{j}(x)\boldsymbol{E}(x)|n\rangle\right| \ll \left|\int_V \mathrm{d}\boldsymbol{r}\operatorname{div}\langle n|\boldsymbol{P}(x)|n\rangle\right|. \tag{38}$$

According to (38) the greater the interaction between the fields (i.e. the greater the second term on the right-hand side of (32)), the less the accuracy to which the quantum processes are described within S-matrix theory.

Making use of the equation which relates the energy density $T^0(x)$ to the momentum density $\boldsymbol{T}(x)$ of the electromagnetic field (see formula (16)), one may easily derive the equation

$$-\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}\boldsymbol{r}\langle 0|T^0(x)|0\rangle$$

$$= \int_V \mathrm{d}\boldsymbol{r}\operatorname{div}\langle 0|\boldsymbol{T}(x)|0\rangle - \sum_\mu g^{\mu\mu}\int_V \mathrm{d}\boldsymbol{r}\langle 0|j_\mu(x)\frac{\partial}{\partial t}A_{\mu,\mathrm{ext}}(x)|0\rangle. \tag{39}$$

Here, the quantised electromagnetic field has not been taken into account. If the condition

$$\operatorname{div}\langle 0|\boldsymbol{T}(x)|0\rangle = 0 \tag{40}$$

is satisfied, the change in energy of the electron-positron field, by (39), is due only to

the work done on the current $j_\mu(x)$ by the applied field. This work is the energy done in the pumping of the vacuum background by an external field. In a homogeneous external field like the field $A_{ext}(t)$ and also in an arbitrary static field equality (40) is valid. Therefore, we may conclude that in the fields mentioned above the electron-positron pairs are not produced, which is in full accord with the results of § 3.

6. Conclusion

Let us summarise the main results of this paper.

Pair production by an external field is the second type of tunnelling. This means that the appearance of vacuum current sources is the necessary condition for pair creation in an external field.

The Klein mechanism of vacuum polarisation in an external field represents the vacuum background deformation which is not accompanied by the creation of real pairs.

The vacuum may be caused by an applied field passing into such an excited state in which the wavefunctions of vacuum electrons contain both negative- and positive-energy components. The excited vacuum structure is governed by the pumping intensity of vacuum vibrations in an external field.

There is, in principle, a possibility of the stimulated emission of the vacuum. In other words, transition of the excited vacuum to the ground state of the vacuum being accompanied by real pair creation and by real photon emission may be induced by an external perturbation.

In some cases the unitarity condition for the S operator is of a rather formal nature and it does not allow one to introduce the probabilistic interpretation. The conventional interpretation of S-matrix scattering theory is not always in agreement with the energy balance equations resulting from the basic equations for quantised fields.

The existence of the vacuum background leads to the concept of the quantum transition probability having no strict definition in quantum electrodynamics. Dirac appears to have been the first to focus attention on the impossibility of determining the transition probability concept in quantum electrodynamics: '... we do not have things we can definitely nail down as probabilities' (Dirac 1964, p 148).

As was noted by Oleinik (1979), because of the vacuum instability in quantum electrodynamics ($\langle 0|S|0\rangle = 0$), the S-matrix elements cannot be interpreted as probability amplitudes for quantum processes. The vacuum state stability principle underlying the conventional formulation of quantum field theory has no meaning as a consequence of the unavoidable appearance of quasi-stationary states in any real physical system (Oleinik and Belousov 1983). The nonzero width of the energy levels of 'dressed' particles means that any real, observable particle is a system which cannot be, in principle, isolated from the surrounding medium—from the vacuum fluctuations of quantised fields (Bogoliubov and Shirkov 1976) and from the classical fields corresponding to the means of observation.

Thus, the fundamental problem arises of constructing the quantum theory of open physical systems. The Hamiltonian formulation of the physical idea of the radical impossibility of isolating a real particle from the surrounding medium is given in Oleinik and Belousov (1983). Here the energy level width is taken as a measure indicating that a particle is not isolated from the medium. The starting points of the theory are the Bogoliubov concept of the quasi-mean and the variational principle applicable to particles with non-zero energy level width.

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